



## Cambridge International AS & A Level

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**MATHEMATICS**

**9709/12**

Paper 1 Pure Mathematics 1

**October/November 2022**

**1 hour 50 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

### INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

### INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

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## 3

1 Points  $A$  and  $B$  have coordinates  $(5, 2)$  and  $(10, -1)$  respectively.

(a) Find the equation of the perpendicular bisector of  $AB$ . [3]

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(b) Find the equation of the circle with centre  $A$  which passes through  $B$ . [3]

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- 2 The first, second and third terms of an arithmetic progression are  $a$ ,  $2a$  and  $a^2$  respectively, where  $a$  is a positive constant.

Find the sum of the first 50 terms of the progression.

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- 3 (a) Find the set of values of  $k$  for which the equation  $8x^2 + kx + 2 = 0$  has no real roots. [2]

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- (b) Solve the equation  $8 \cos^2 \theta - 10 \cos \theta + 2 = 0$  for  $0^\circ \leq \theta \leq 180^\circ$ . [3]

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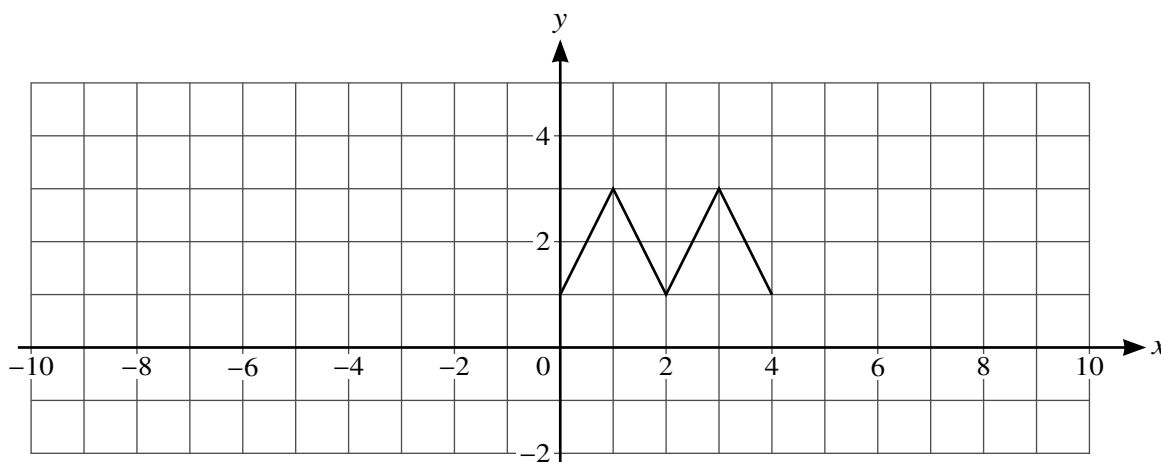
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5 The graph with equation  $y = f(x)$  is transformed to the graph with equation  $y = g(x)$  by a stretch in the  $x$ -direction with factor 0.5, followed by a translation of  $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ .

(a) The diagram below shows the graph of  $y = f(x)$ .

On the diagram sketch the graph of  $y = g(x)$ . [3]



(b) Find an expression for  $g(x)$  in terms of  $f(x)$ . [2]

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6 The equation of a curve is  $y = 4x^2 + 20x + 6$ .

(a) Express the equation in the form  $y = a(x + b)^2 + c$ , where  $a$ ,  $b$  and  $c$  are constants. [3]

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(b) Hence solve the equation  $4x^2 + 20x + 6 = 45$ . [3]

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- (c) Sketch the graph of  $y = 4x^2 + 20x + 6$  showing the coordinates of the stationary point. You are not required to indicate where the curve crosses the  $x$ - and  $y$ -axes. [3]

7 (a) Prove the identity  $\frac{\sin \theta}{\sin \theta + \cos \theta} + \frac{\cos \theta}{\sin \theta - \cos \theta} \equiv \frac{\tan^2 \theta + 1}{\tan^2 \theta - 1}$ . [3]

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- (b) Hence find the exact solutions of the equation  $\frac{\sin \theta}{\sin \theta + \cos \theta} + \frac{\cos \theta}{\sin \theta - \cos \theta} = 2$  for  $0 \leq \theta \leq \pi$ . [4]

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8 The equation of a curve is such that  $\frac{dy}{dx} = 3x^{\frac{1}{2}} - 3x^{-\frac{1}{2}}$ . The curve passes through the point (3, 5).

(a) Find the equation of the curve. [4]

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(b) Find the  $x$ -coordinate of the stationary point. [2]

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(c) State the set of values of  $x$  for which  $y$  increases as  $x$  increases. [1]

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9 Functions  $f$  and  $g$  are defined by

$$f(x) = x + \frac{1}{x} \quad \text{for } x > 0,$$

$$g(x) = ax + 1 \quad \text{for } x \in \mathbb{R},$$

where  $a$  is a constant.

(a) Find an expression for  $gf(x)$ . [1]

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(b) Given that  $gf(2) = 11$ , find the value of  $a$ . [2]

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(c) Given that the graph of  $y = f(x)$  has a minimum point when  $x = 1$ , explain whether or not  $f$  has an inverse. [1]

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It is given instead that  $a = 5$ .

- (d) Find and simplify an expression for  $g^{-1}f(x)$ . [3]

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- (e) Explain why the composite function  $fg$  cannot be formed. [1]

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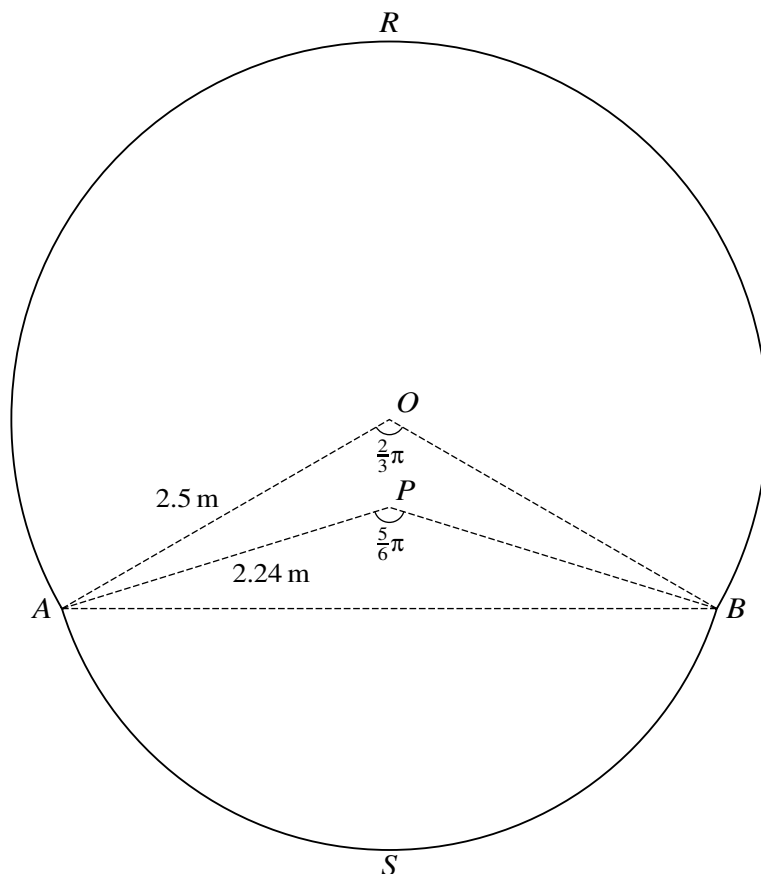
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The diagram shows a cross-section  $RASB$  of the body of an aircraft. The cross-section consists of a sector  $OARB$  of a circle of radius 2.5 m, with centre  $O$ , a sector  $PASB$  of another circle of radius 2.24 m with centre  $P$  and a quadrilateral  $OAPB$ . Angle  $AOB = \frac{2}{3}\pi$  and angle  $APB = \frac{5}{6}\pi$ .

- (a) Find the perimeter of the cross-section  $RASB$ , giving your answer correct to 2 decimal places.

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(b) Find the difference in area of the two triangles  $AOB$  and  $APB$ , giving your answer correct to 2 decimal places. [2]

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(c) Find the area of the cross-section  $RASB$ , giving your answer correct to 1 decimal place. [3]

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- 11 (a) Find the coordinates of the minimum point of the curve  $y = \frac{9}{4}x^2 - 12x + 18$ . [3]

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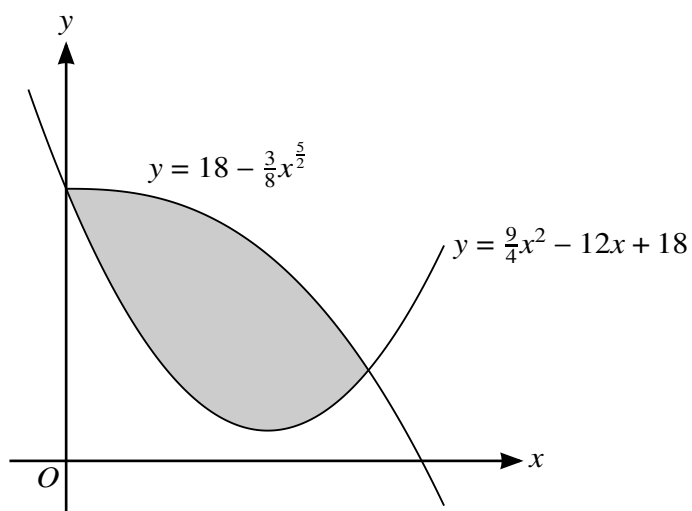
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The diagram shows the curves with equations  $y = \frac{9}{4}x^2 - 12x + 18$  and  $y = 18 - \frac{3}{8}x^5$ . The curves intersect at the points  $(0, 18)$  and  $(4, 6)$ .

- (b) Find the area of the shaded region. [5]

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- (c) A point  $P$  is moving along the curve  $y = 18 - \frac{3}{8}x^{\frac{5}{2}}$  in such a way that the  $x$ -coordinate of  $P$  is increasing at a constant rate of 2 units per second.

Find the rate at which the  $y$ -coordinate of  $P$  is changing when  $x = 4$ . [3]

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